Converting from rocket lateral forces to gimbal servos rotation angles We consider a rocket with a rectangular coordinate system with x-axis in the longitudinal axis and the origin on the center of rotation of the motor. Each servo produces a rotation, but the inner one is mounted on the outer one. The thrust rotates first in rocket axis and with the outer servo and the second one rotates with respect to the already rotated coordinate axis. This is represented as two consecutive rotations, first, with respect to z and secondly with respect to y.

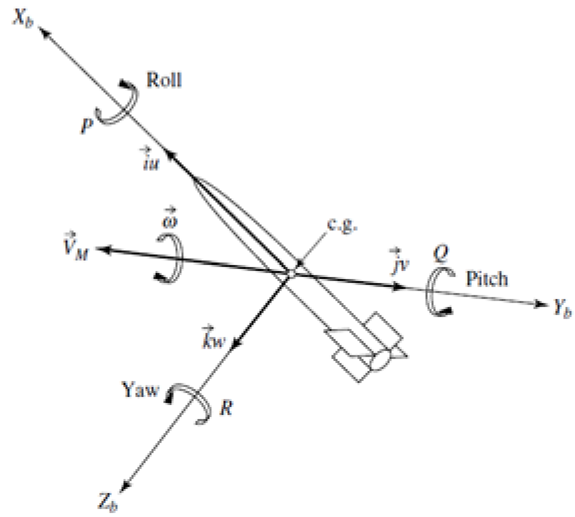

Figure 1: Rocket coordinate system

$$\boldsymbol{R_y} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \tag{1}$$

$$\boldsymbol{R_z} = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\boldsymbol{R} = \boldsymbol{R_y}\boldsymbol{R_z} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\psi c\theta & -c\theta s\psi & s\theta \\ s\psi & c\psi & 0 \\ -c\psi s\theta & s\theta s\theta & c\theta \end{bmatrix} \tag{3}$$

If we then rotate a unit thrust vector we obtain,

$$\boldsymbol{R} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -c\psi c\theta \\ -s\psi \\ c\psi s\theta \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{4}$$

$$\begin{cases} -s\psi = u_y \\ c\psi s\theta = u_z \end{cases} \tag{5}$$

We are interested in the inverse relationship. If small angles are assumed, relationships and linear and uncoupled.

$$\begin{cases} \psi = s^{-1}\left(-u_y\right) \simeq -u_y \\ \theta = s^{-1}\left(\frac{u_z}{\sqrt{1-u_y^2}}\right) \simeq u_z \quad \left(c\psi = \sqrt{1-s^2\psi} = \sqrt{1-u_y^2}\right) \end{cases} \tag{6}$$