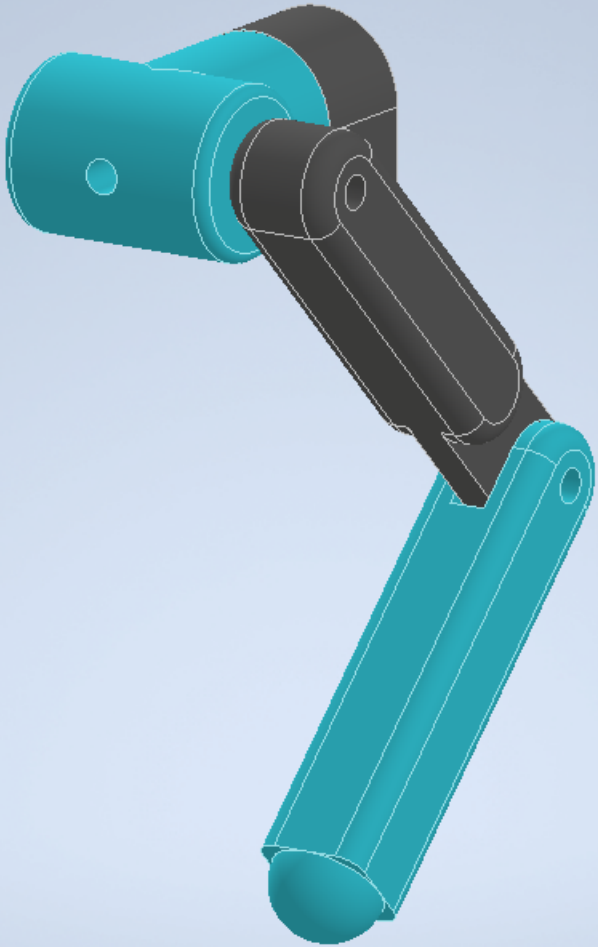
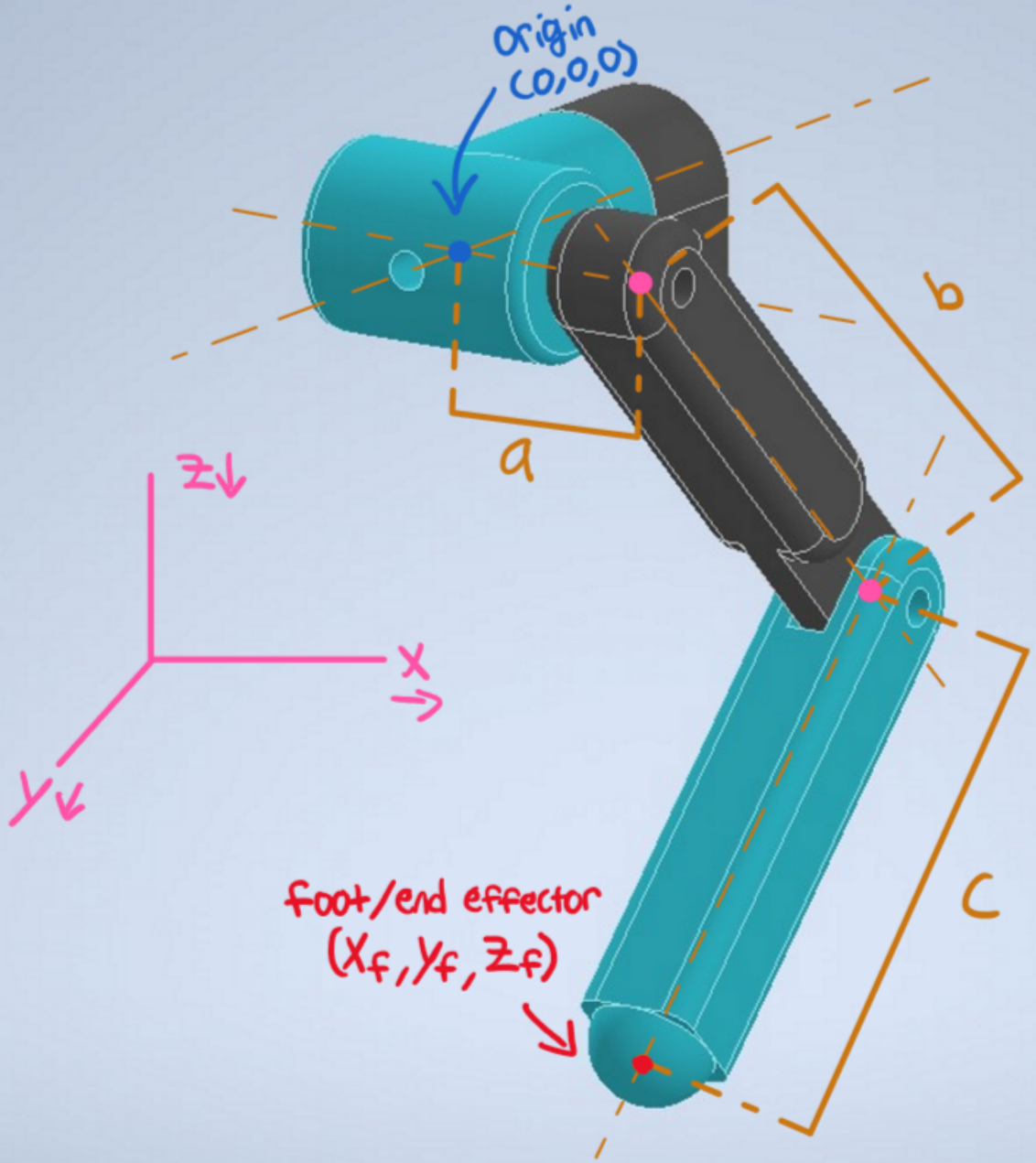
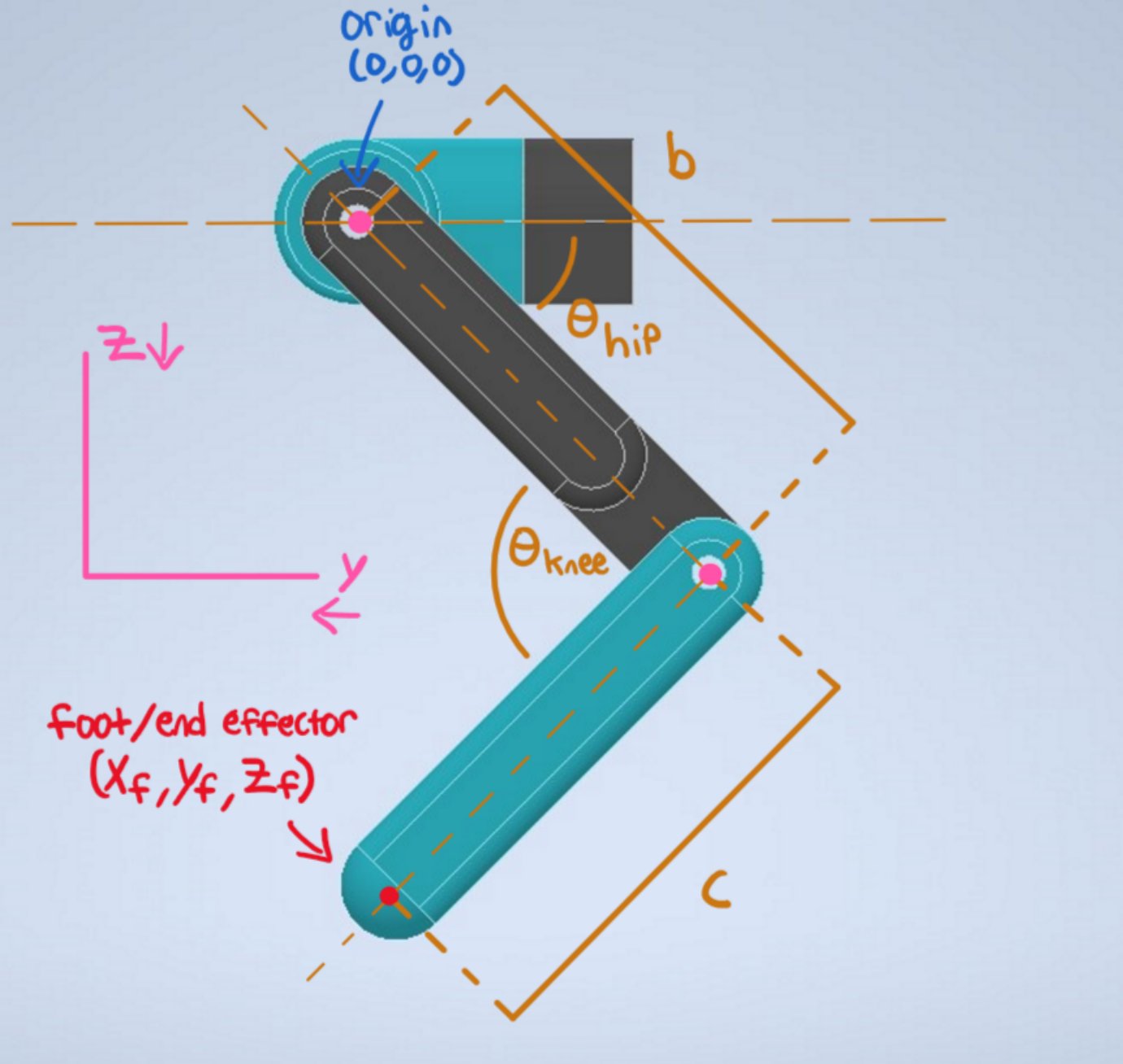
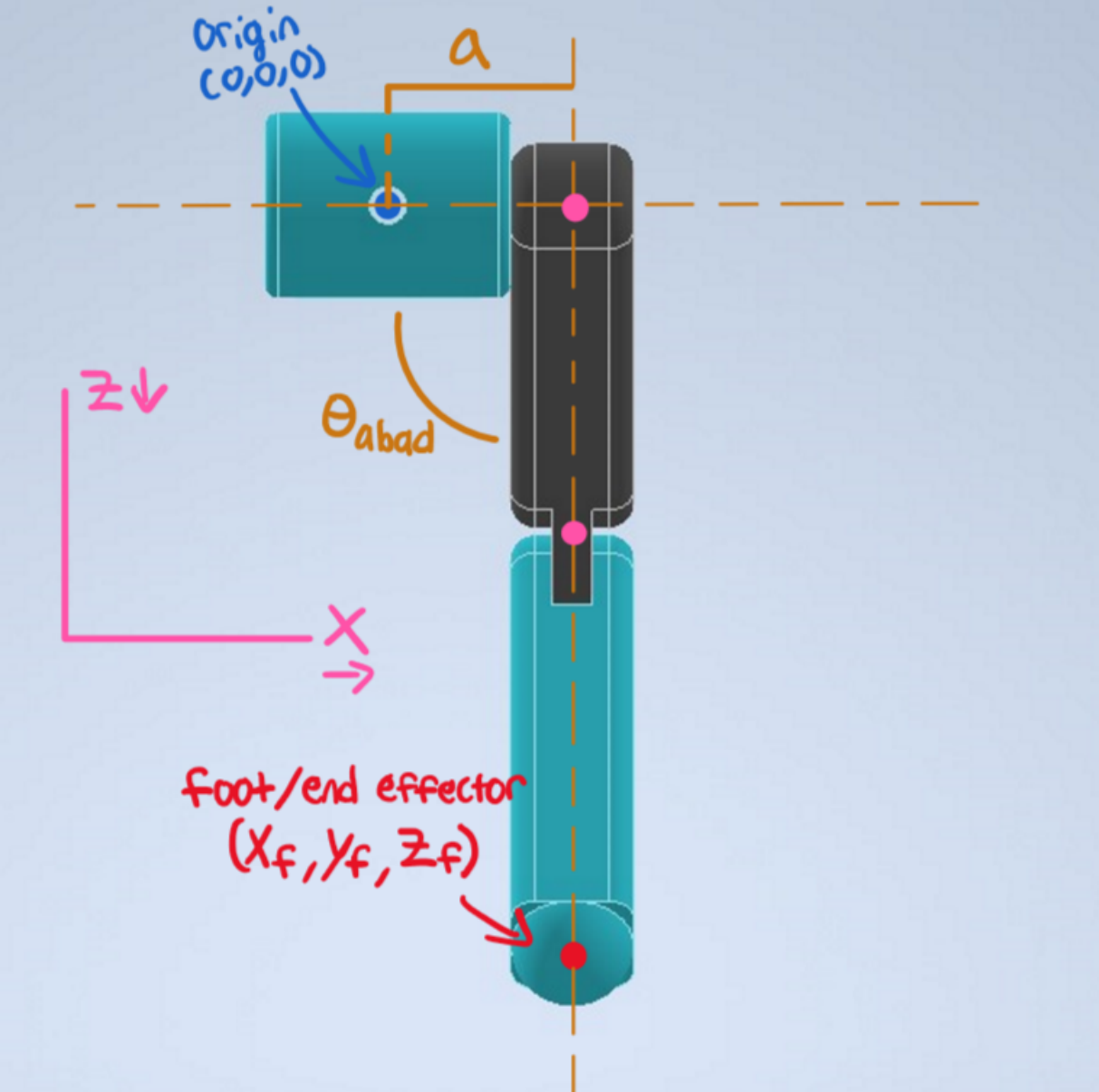
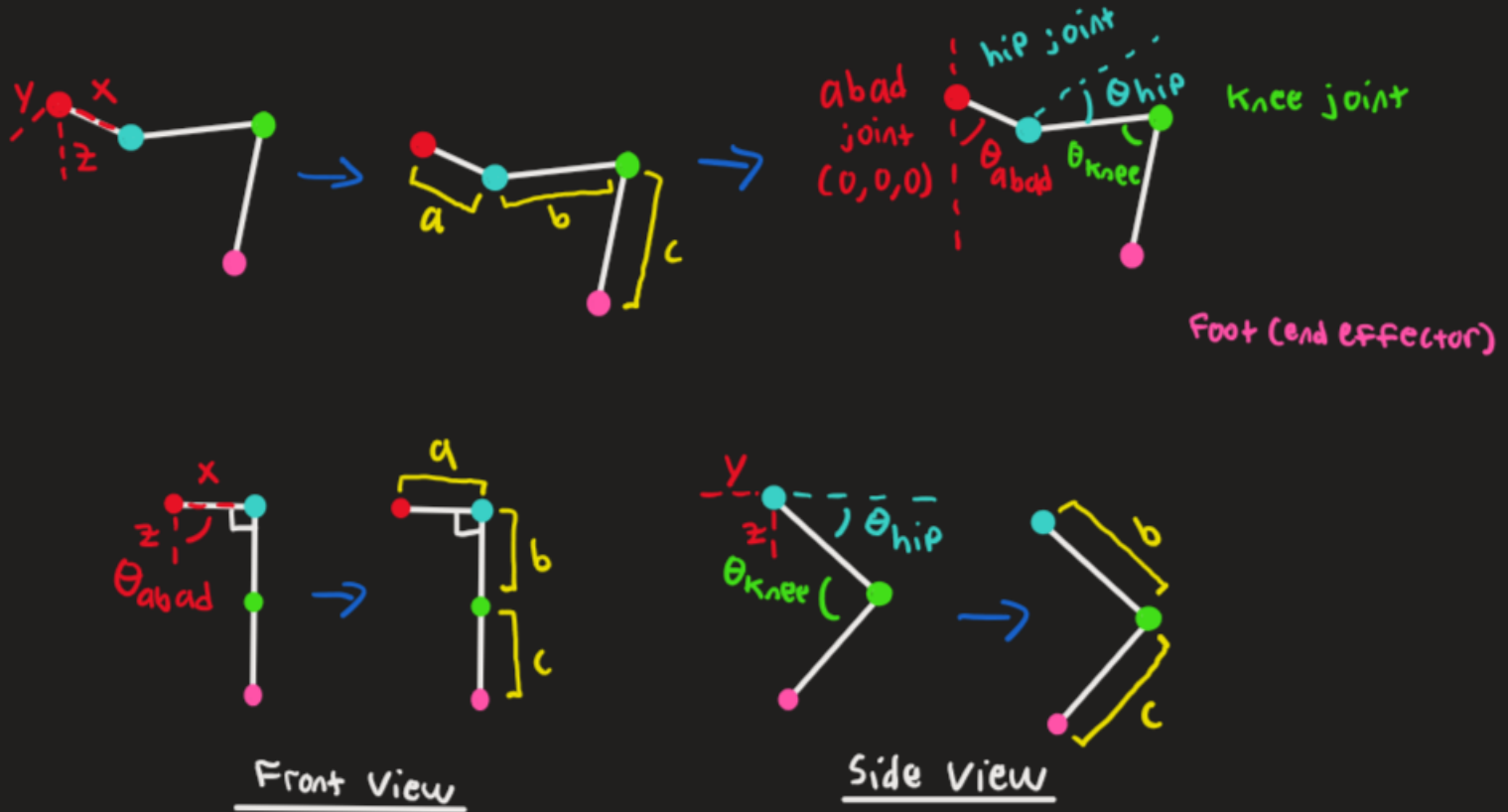



origin
(0,0,0)
a
b
c
Z
X
Y
foot/end effector
(X_f, Y_f, Z_f)


origin
(0,0,0)
b
θhip
z
y
θknee
foot/end effector
(Xf, Yf, Zf)
c


origin
(0,0,0)
a
Z
θ_{abad}
X
foot/end effector
(X_f, Y_f, Z_f)



defined	unknowns
foot $= (x_f, y_f, z_f)$	$\theta_{abad}, \theta_{hip}, \theta_{knee}$
adab $= (0,0,0)$	
a, b, c	

1. Solve for θ_{knee}

$$l = \sqrt{x_f^2 + y_f^2 + z_f^2}$$

$$l^2 = L = m^2 + a^2$$

$$m = \sqrt{L - a^2}$$

Law of cosines

$$\theta_{knee} = \cos^{-1}\left(\frac{b^2 + c^2 - m^2}{2bc}\right) =$$

$$\theta_{knee} = \cos^{-1}\left(\frac{a^2 + b^2 + c^2 - L}{2bc}\right)$$

2. Solve for θ_{hip}

$$\theta_{hip} = 90 - \theta_1 - \theta_2$$

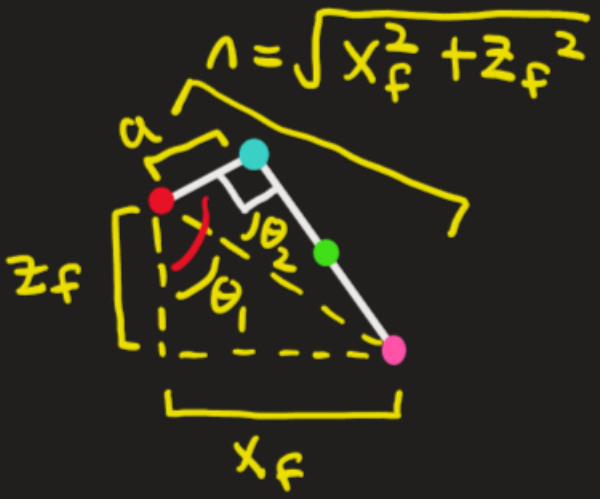
$$\theta_1 = \sin^{-1}\left(\frac{-Y_f}{m}\right) = \sin^{-1}\left(\frac{-Y_f}{\sqrt{L-a^2}}\right)$$

Law of Cosines

$$\theta_2 = \cos^{-1}\left(\frac{b^2+m^2-c^2}{2bm}\right) = \cos^{-1}\left(\frac{b^2-a^2-c^2+L}{2b\sqrt{L-a^2}}\right)$$

$$\theta_{hip} = 90 - \sin^{-1}\left(\frac{-Y_f}{\sqrt{L-a^2}}\right) - \cos^{-1}\left(\frac{b^2-a^2-c^2+L}{2b\sqrt{L-a^2}}\right)$$

3. Solve for θ_{abad}



$$\theta_{adab} = \theta_1 + \theta_2$$

$$\theta_1 = \tan^{-1}\left(\frac{x_f}{z_f}\right)$$

$$\theta_2 = \cos^{-1}\left(\frac{a}{\sqrt{x_f^2 + z_f^2}}\right)$$

$$\theta_{abad} = \tan^{-1}\left(\frac{x_f}{z_f}\right) + \cos^{-1}\left(\frac{a}{\sqrt{x_f^2 + z_f^2}}\right)$$

Given

End effector (foot) = (x_f, y_f, z_f)

and

a, b, and c

- $L = x_f^2 + y_f^2 + z_f^2$
- $\theta_{abad} = \tan^{-1}\left(\frac{x_f}{z_f}\right) + \cos^{-1}\left(\frac{a}{\sqrt{x_f^2 + z_f^2}}\right)$
- $\theta_{hip} = 90 - \sin^{-1}\left(\frac{-y_f}{\sqrt{L-a^2}}\right) - \cos^{-1}\left(\frac{b^2 - a^2 - c^2 + L}{2b\sqrt{L-a^2}}\right)$
- $\theta_{knee} = \cos^{-1}\left(\frac{a^2 + b^2 + c^2 - L}{2bc}\right)$