Mid z position of platform $z_{mid} := 20mm$

Platform displacement

| | | | |
|---|---|---|---|
| Surge (x) | $x := 0.5mm$ | Roll (rx) | $\varphi := 0deg$ |
| Sway (y) | $y := 0mm$ | Pitch (ry) | $\theta := 0deg$ |
| Heave (z) | $z := 0mm$ | Yaw (rz) | $\psi := 0deg$ |

roll rotation matrix
$$R_x := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

< Only need to calc 4 values in each table & range of valid angles is going to be limited, so can precalculate a subset of cos & sin values. Also need to develop fixed point sizing to maximise resolution for 16bit operations

Pitch rotation matrix
$$R_y := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Yaw rotation matrix
$$R_z := \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Combined rotation matrix
$$R_B := R_z \cdot R_y \cdot R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

leg: 0 1 2 3 4 5

Base anchor point co-ordinates (local)
$$b := \begin{pmatrix} -3 & 3 & 25.75 & 22.75 & -22.75 & -25.75 \\ 28 & 28 & -11.4 & -16.6 & -16.6 & -11.4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot mm$$

axis
x
y
z

Platform anchor point co-ordinates (local)
$$p := \begin{pmatrix} -5.5 & 5.5 & 7.5 & 2 & -2 & -7.5 \\ 5.48 & 5.48 & 2.02 & -7.5 & -7.5 & 2.02 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot mm$$

point of interest offset from platform coordinate
$$offset := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \cdot mm$$

< relative to the actual position of interest on the platform or the end of the item being actuated. This then compensates for the translation effect of rotation.

Translation array
$$T := \begin{pmatrix} x + offset_0 \\ y + offset_1 \\ z + z_{mid} + offset_2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ 20 \end{pmatrix} \cdot mm$$

Leg length vectors

$$vec_{leg} := \left| \begin{array}{l} \text{for } i \in 0..5 \\ \quad l^{\langle i \rangle} \leftarrow T + R_B \cdot \left(p^{\langle i \rangle} - offset\right) - b^{\langle i \rangle} \\ \text{return } l \end{array} \right.$$

$$vec_{leg} = \begin{pmatrix} -2 & 3 & -17.75 & -20.25 & 21.25 & 18.75 \\ -22.52 & -22.52 & 13.42 & 9.1 & 9.1 & 13.42 \\ 20 & 20 & 20 & 20 & 20 & 20 \end{pmatrix} \cdot mm$$

leg mid length

$$leg_{mid} := 30mm$$

get length from vectors

$$leg_{len} := \left| \begin{array}{l} \text{for } i \in 0..5 \\ \quad l_{len}{}^{\langle i \rangle} \leftarrow \sqrt{\left(vec_{leg_{0,i}}\right)^2 + \left(vec_{leg_{1,i}}\right)^2 + \left(vec_{leg_{2,i}}\right)^2} \\ \text{return } l_{len} - leg_{mid} \end{array} \right.$$

<- need to record the previous sign for movement to account for backlash in system

$$leg_{len} = \left(0.185 \quad 0.268 \quad -0.081 \quad -0.119 \quad 0.568 \quad 0.523\right) \cdot mm$$

max angle $max := 5$ deg

$div := 0.1$ deg

$uint(x) := \left| x - mod(x,1) \right|$

pre-calculate the sin and cos values for valid range of angles at

$$sin\_table := \left| \begin{array}{l} \text{for } i \in 0 .. \frac{max}{div} \\ \quad table_i \leftarrow \sin[(i \cdot div)deg] \\ \text{return } table \end{array} \right.$$

$$cos\_table := \left| \begin{array}{l} \text{for } i \in 0 .. \frac{max}{div} \\ \quad table_i \leftarrow \cos[(i \cdot div)deg] \\ \text{return } table \end{array} \right.$$

sin_table =

| | 0 |
|---|---|
| 0 | 0 |
| 1 | 0.001745 |
| 2 | 0.003491 |
| 3 | 0.005236 |
| 4 | 0.006981 |
| 5 | 0.008727 |
| 6 | 0.010472 |
| 7 | 0.012217 |
| 8 | 0.013962 |
| 9 | 0.015707 |
| 10 | 0.017452 |
| 11 | 0.019197 |
| 12 | 0.020942 |
| 13 | 0.022687 |
| 14 | 0.024432 |
| 15 | ... |

$\sin(3deg) = 0.05234$

cos_table =

| | 0 |
|---|---|
| 0 | 1 |
| 1 | 0.999998 |
| 2 | 0.999994 |
| 3 | 0.999986 |
| 4 | 0.999976 |
| 5 | 0.999962 |
| 6 | 0.999945 |
| 7 | 0.999925 |
| 8 | 0.999903 |
| 9 | 0.999877 |
| 10 | 0.999848 |
| 11 | 0.999816 |
| 12 | 0.999781 |
| 13 | 0.999743 |
| 14 | 0.999701 |
| 15 | ... |

$$s(x) := \left| \begin{array}{l} lower \leftarrow uint\left(\frac{x}{div}\right) \\ \text{return } sin\_table_{lower} \end{array} \right.$$

$$uint\left(\frac{3}{div}\right) = 30$$

$$c(x) := \left| \begin{array}{l} lower \leftarrow uint\left(\frac{x}{div}\right) \\ \text{return } cos\_table_{lower} \end{array} \right.$$

$pitch := \frac{\theta}{deg} = 0$ $\quad roll := \frac{\varphi}{deg} = 0$ $\quad yaw := \frac{\psi}{deg} = 0$

$$R_B := \begin{pmatrix} c(\psi) \cdot c(\theta) & -s(\psi) \cdot c(\varphi) + c(\psi) \cdot s(\theta) \cdot s(\varphi) & s(\psi) \cdot s(\varphi) + c(\psi) \cdot s(\theta) \cdot c(\varphi) \\ s(\psi) \cdot c(\theta) & c(\psi) \cdot c(\varphi) + s(\psi) \cdot s(\theta) \cdot s(\varphi) & -c(\psi) \cdot s(\varphi) + s(\psi) \cdot s(\theta) \cdot c(\varphi) \\ -s(\theta) & c(\theta) \cdot s(\varphi) & c(\theta) \cdot c(\varphi) \end{pmatrix}$$

Combined rotation matrix

$$R_B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

check that formula is correct

$$leg\_index := 0$$

Translation array

$$T := \begin{pmatrix} x + offset_0 \\ y + offset_1 \\ z + z_{mid} + offset_2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ 20 \end{pmatrix} \cdot mm$$

leg 0, platform co-ordinates

$$P := p^{\langle leg\_index \rangle} - offset = \begin{pmatrix} -5.5 \\ 5.48 \\ 0 \end{pmatrix} \cdot mm$$

leg 0, base co-ordinates

$$b := b^{\langle leg\_index \rangle} = \begin{pmatrix} -3 \\ 28 \\ 0 \end{pmatrix} \cdot mm$$

$$leg := \begin{bmatrix} T_0 + c(yaw)\cdot c(pitch)\cdot P_0 + (-s(yaw)\cdot c(roll) + c(yaw)\cdot s(pitch)\cdot s(roll))\cdot P_1 \dots \\ + \left[(s(yaw)\cdot s(roll) + c(yaw)\cdot s(pitch)\cdot c(roll))\cdot P_2 - b_0\right] \\ T_1 + s(yaw)\cdot c(pitch)\cdot P_0 + (c(yaw)\cdot c(roll) + s(yaw)\cdot s(pitch)\cdot s(roll))\cdot P_1 \dots \\ + \left[(-c(yaw)\cdot s(roll) + s(yaw)\cdot s(pitch)\cdot c(roll))\cdot P_2 - b_1\right] \\ T_2 + -s(pitch)\cdot P_0 + c(pitch)\cdot s(roll)\cdot P_1 + c(pitch)\cdot c(roll)\cdot P_2 - b_2 \end{bmatrix}$$

leg vector

$$leg = \begin{pmatrix} -2 \\ -22.52 \\ 20 \end{pmatrix} \cdot mm$$

$$leg\_len := \sqrt{(leg_0)^2 + (leg_1)^2 + (leg_2)^2} = 30.185 \cdot mm$$

$$leg\_change := leg\_len - leg_{mid} = 0.185 \cdot mm$$