Nodal Analysis applied to Active circuits

Active circuits can be solved by many different techniques. Using those techniques often require dividing a circuit into multiple separate blocks of forward and feedback functions. Sometimes it is a very difficult task to determine forward and feedback paths of a particular linear active circuit. Any linear circuit can be solved as a combination of linear equations. These equations can be written around voltage nodes.Nodal analysis is widely used to analyze passive netwoks. The following figure shows a basic resistive network.

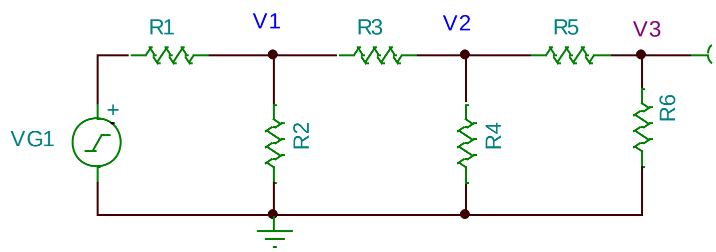

Figure 1

The circuit has three nodes, which can be described in three linear equations.

$$V1\left(\frac{1}{R1}+\frac{1}{R2}+\frac{1}{R3}\right)-\frac{1}{R1}VG1-\frac{1}{R3}V2=0$$

$$V2\left(\frac{1}{R3}+\frac{1}{R4}+\frac{1}{R5}\right)-\frac{1}{R3}V1-\frac{1}{R5}V3=0$$

$$V3\left(\frac{1}{R5}+\frac{1}{R6}\right)-\frac{1}{R5}V2=0$$

Solution for voltages V1, V2, and V3 can be found using linear algebra techniques.

A basic nodal analysis can be easily applied to any linear active circuit. For example lets take alook at a second order low-pass filter.

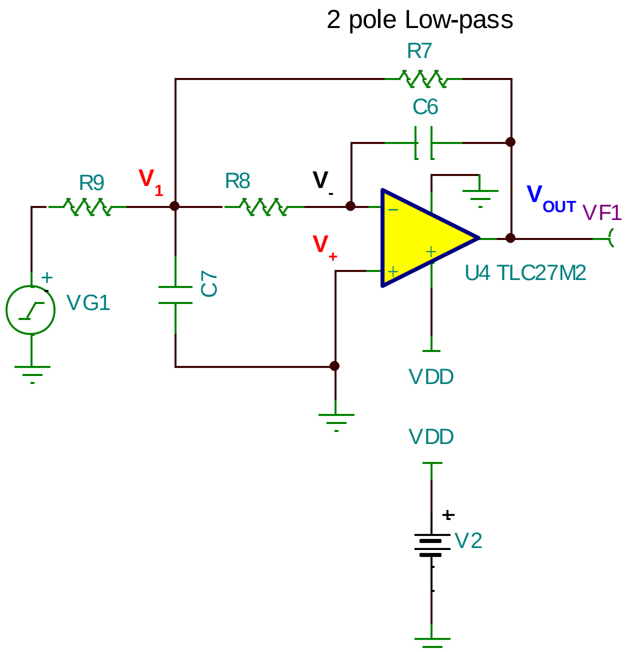

Figure 2

Using nodal analysis the following can be obtained at nodes V1 and V_-:

$$V1\left(\frac{1}{R9}+\frac{1}{R8}+\frac{1}{R7}+sC7\right)-\frac{1}{R9}VG1-\frac{1}{R7}V_{OUT}-\frac{1}{R8}V_-=0 \qquad (1)$$

$$V_-\left(\frac{1}{R8}+sC6\right)-sC6\,V_{OUT}-\frac{1}{R8}V1=0 \qquad (2)$$

Assuming ideal operational amplifier, the following is true:

$$V_- = V_+ = 0$$

In equation (1) the term that contains 1/R8 V_- is zero.

$$V1\left(\frac{1}{R9}+\frac{1}{R8}+\frac{1}{R7}+sC7\right)-\frac{1}{R9}VG1-\frac{1}{R7}V_{OUT}=0 \qquad (3)$$

In equation (2) the term that contains $V_-\left(\frac{1}{R8}+sC6\right)$ is also zero.

$$-sC6\,V_{OUT}-\frac{1}{R8}V1=0 \qquad (4)$$

The first term in equation (3) can be presented as follows:

$$V1\left(\frac{1}{R9}+\frac{1}{R8}+\frac{1}{R7}+sC7\right)=V1\,\frac{R7\,R8+R7\,R9+R8\,R9+R9\,R8\,R7\,sC7}{R9\,R8\,R7} \qquad (5)$$

Substitute (5) into equation (1), we will get:

$$V1\ \frac{R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7}{R9\ R8\ R7}\ -\frac{1}{R9}VG1-\ \frac{1}{R7}V_{OUT}\ =0 \qquad (6)$$

From equation (4) we can determine V_{OUT}.

$$V_{OUT}=-\ \frac{1}{sC6\ R8}\ V1 \qquad (7)$$

Solving for V1 in equation (6).

$$V1=\ \frac{R9\ R8\ R7}{R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7}\ (\frac{1}{R9}VG1+\ \frac{1}{R7}V_{OUT}\ ) \qquad (8)$$

Substituting equation (8) into equation (7) we will get.

$$V_{OUT}=-\ \frac{1}{sC6\ R8}\ \frac{R9\ R8\ R7}{R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7}\ (\frac{1}{R9}VG1+\ \frac{1}{R7}V_{OUT}\ ) \qquad (9)$$

Simplifying

$$V_{OUT}(1+\frac{R9}{sC6(R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7)})=-\ \frac{R7}{sC6(R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7)}\ VG1$$

$$V_{OUT}(\frac{sC6(R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7)+R9}{sC6(R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7)})=-\ \frac{R7}{sC6(R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7)}\ VG1$$

$$V_{OUT}=-\ \frac{R7}{sC6(R7\ R8+R7\ R9+R8\ R9+R9\ R8\ R7\ sC7)+R9}\ VG1 \qquad (10)$$

The above equation can be rewritten as:

$$V_{OUT}=-\ \frac{\frac{1}{R9\ R8\ C7\ C6}}{s^2+s\ \frac{C6\ R7\ R8+C6\ R7\ R9+C6\ R8\ R9}{R9\ R8\ R7\ C7\ C6}+\ \frac{1}{R8\ R7\ C7\ C6}}\ VG1$$

$$V_{OUT}=-\ \frac{\frac{R7}{R8}}{s^2R8\ R7\ C7\ C6+s\ \frac{C6\ (R7\ R8+\ R7\ R9+R8\ R9)}{R9}+\ 1}\ VG1$$

Where gain of the circuit is: $A_0=-\frac{R7}{R8}$

$$a_1=\ \frac{C6\ (R7\ R8+\ R7\ R9+R8\ R9)}{R9}$$

$$b_1=R8\ R7\ C7\ C6$$

$$\omega = \frac{1}{\sqrt{R8\ R7\ C7\ C6}}$$

Conclusion: Nodal analysis can be used on any type of linear circuit. Circuit nodes can be easily identified from a circuit diagram and a circuit solution becomes a straight forward mathematical manipulation.